# Delhi Public School Guwahati

"Under the aegis of the Delhi Public School Society, Delhi"

## SUMMER VACATION HOLIDAY HOME-WORK

## CLASS: XII

## SUBJECT: MATHEMATICS

1. Construct a 2×3 matrix B=[$b_{ij}$] whose element $b_{ij}$ is given by $|2i-3j|$.
2. Write the number of all possible matrices of order 2×3 with each entry 5, 7 or 8.
3. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix}=\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix}+\begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w.
4. Find a matrix A such that 2A − 3B + 5C = 0, where B = $\begin{bmatrix} 5 & 3 & 1 \\ 2 & 0 & 4 \end{bmatrix}$ and C = $\begin{bmatrix} 4 & 3 & 2 \\ 0 & -5 & -1 \end{bmatrix}$.
5. If A = [1,−2,5], B = [3,0,−4] and C = [−2,7,0], then find 2A + B −2C.
6. Find x + y + z, if $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.
7. If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then find the value of $A^2-4A-5I$.
8. Write the following as a single matrix: $\begin{bmatrix} 3 & 2 & 5 \\ 7 & -4 & 0 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & -1 \\ 3 & 5 \end{bmatrix}-\begin{bmatrix} 7 & -8 \\ 5 & 9 \end{bmatrix}$.
9. If A = $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$, verify that $(AB)' = B'A'$.
10. If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{bmatrix}$ is a matrix satisfying $AA' = 9I_3$, find x.
11. Solve the following system of linear equations by Matrix rule:

(i) $\begin{aligned} 6x+y-3z-5&=0 \\ x+3y-2z&=5 \\ 2x+4z-8&=-7 \end{aligned}$

(ii) $\begin{aligned} \frac{2}{x}+\frac{3}{y}&=2 \\ \frac{5}{x}+\frac{8}{y}&=\frac{31}{6} \end{aligned}$

12. Prove that $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\frac{\pi}{4}-\frac{x}{2},\ \pi<x<\frac{3\pi}{2}$
13. If $y=\cot^{-1}\left(\sqrt{\cos x}\right)-\tan^{-1}\left(\sqrt{\cos x}\right)$, prove that $\sin y=\tan^2\frac{x}{2}$.

14. Prove that $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

15. Solve the following equations for x: $\cos\left(\tan^{-1}x\right)=\sin\left(\cot^{-1}\frac{3}{4}\right)$

16. Prove that $\sin\left(\cot^{-1}\left(\cos\left(\tan^{-1}x\right)\right)\right)=\sqrt{\frac{x^2+1}{x^2+2}}$

17. If $f\left(x\right)=\frac{4^x}{4^x+2}$, then find the value of $f\left(x\right)+f\left(1-x\right)$. (Ans: 1)

18. Let N be the set of all natural numbers and R be the relation on N × N defined by $\left(a,b\right)R\left(c,d\right)$ if $ad\left(b+c\right)=bc\left(a+d\right)$.Show that R is an equivalence relation.

19. Consider $f:R_+\to\left[-9,\infty\right)$ given by $f\left(x\right)=5x^2+6x-9$. Prove that f is one-one and onto.

20. Show that the function $f\left(x\right)=\left|x-3\right|,\ x\in R$ is continuous but not differentiable at $x=3$.

21. For what value of k, the following function is continuous at $x=0$.

$$f\left(x\right)=\begin{cases}\frac{1-\cos 4x}{8x^2}, & if \quad x\neq 0\\ k \quad , & if \quad x=0\end{cases}$$ (Ans: 1)

22. For what value of k, the following function is continuous at $x=0$.

$$f\left(x\right)=\begin{cases}\frac{1-\cos 4x}{x^2} \quad , & if \quad x<0\\ k \quad , & if \quad x=0\\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} \quad , & if \quad x>0\end{cases}$$ (Ans: 8)

23. Discuss the continuity of the function at $x=0$

$$f\left(x\right)=\begin{cases}\frac{\sin 3x}{\tan 2x} \quad , & if \quad x<0\\ \frac{\log\left(1+3x\right)}{e^{2x}-1}, & if \quad x>0\\ \frac{3}{2} \quad , & if \quad x=0\end{cases}$$

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